A comparison of ensemble- and volume-averaged bubbly flow models

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Keywords: Bubble dynamics, cavitation, multiphase flow modeling, compressible flow

Abstract

We compare volume- and ensemble-averaged bubbly flow models. Volume-averaging is a deterministic process for which bubbles are represented in a Lagrangian framework as advected particles, each sampled from a distribution of equilibrium bubble sizes. Ensemble-averaging instead uses mixture-averaged equations in an Eulerian reference frame for the associated bubble properties, each represented by bins of the equilibrium distribution. In both cases, bubbles are modeled as spherical with dynamics governed by the Keller-Miksis equation. Computationally, there are tradeoffs between these two approaches. Here, we simulate an acoustically excited dilute bubble screen and compare the computational efficiency of the two approaches.

Introduction

We consider complex cavitating bubbly flows, where bubbles can oscillate, expand significantly, and collapse violently. Notably, the multiphase bulk flow is sensitive to individual bubble motion; the shockwaves emitting from a cavitation event are often comparable to those in the bulk, and even just a few bubbles are sufficient to modify larger-scale pressure waves (Mettin and Lauterborn 2003). Such flows often occur naturally, such as during mantis shrimp strikes (Bauer 2004), and in applications, including shockwave lithotripsy (Coleman et al. 1987).

Unfortunately, analyzing bubbly flow is challenging primarily due to the vast range of length scales involved, from the radius of a bubble-nuclei to the size of bubble clouds and turbulent structures. This makes fully resolved computer simulations prohibitive. Instead, modeling is required to represent the flow dynamics. The first models for bubbly flows provided theories for linear scattering (Foldy 1945) and nonlinear oscillatory systems (Iordanskii 1960). Since then, most models have been broadly classified as either ensemble averaged- (Zhang and Prosperetti 1994) or volume averaged (Commander and Prosperetti 1989). We focus on a specific example of each model and assess their relative utility and advantages.

Bubbly-flow models

In both cases, the mixture-averaged flow equations and bubbles are represented as sub-grid features interacting with the flow. However, the bubbles are tracked and coupled to the liquid phase differently. In volume-averaged models, the volume of gas per-unit-volume of the mixture is obtained locally for each computational cell by projecting the volume of bubbles onto the grid. The disturbances induced by the bubbles on the flow field is determined by evaluating the background and bubble flow potentials individually (Fuster and Colonius 2011). The ensemble-averaged approach instead evaluates the statistically-averaged mixture dynamics by assuming a large number of isotropically scattered bubbles as disperse (Ando et al. 2011).

Besides algorithmic differences, there are also important differences in the fundamental assumptions. In the volume-averaged case, for the mixture to be considered homogeneous and wave structure to be resolved, the length scale of the control volume must be larger than mean bubble spacing but smaller than the mixture wavelength. Ensemble-averaged models are not beholden to this assumption, though ultimately the separation of scales is still assumed for model closure; so long as this scale separation is obeyed, ensemble- and volume-averaging are equivalent.Unlike the volume-averaged approach, ensemble-averaged models assume there are no interactions between bubbles, except through their effect on the mixture-averaged flow. In the present study, we assume that the bubbles are spherical, their number is conserved, and they advect at the local liquid velocity, though in principle these assumptions can be relaxed with appropriate model extensions.

The continuum ensemble-averaged equations follow from Zhang and Prosperetti (1994) . The mixture pressure p is a linear combination of the liquid pressure and the phase interactions. All mean bubble variables are sampled with respect to an assumed known bubble size distribution. In addition to the phase-averaged equations, the void fraction is also transported with source terms accounting for volume change. The volume-averaged equations are instead cast in terms of the liquid phase (Fuster and Colonius 2011). Each bubble is located in space and tracked as a Lagrangian point (Maeda and Colonius 2018). The continuous void fraction field is computed by smearing the bubble volume with a Gaussian regularization kernel. In both cases, the liquid pressure is modeled according to a stiffened gas equation of state.

The mixture-averaged equations are closed by the bubble dynamic equations. Bubble oscillations are forced by the far field pressure and modeled by the Keller–Miksis equation. The internal bubble pressure is tracked independently according to Ando et al. (2011). Mass transfer of the bubble contents follows the reduced model of Preston et al. (2007). This model includes thermal effects, viscous and acoustic damping, and phase change.

Numerical method

Our numerical scheme generally follows that of Coralic and Colonius (2006). The models are written in a quasi-conservation form. The spatial discretization is a finite-volume grid and the flux variables are evaluated within each cell-centered volume. We reconstruct the primitive variables at the finite-volume-cell faces via a WENO5 scheme and use the HLLC Riemann solver to compute the fluxes. The time derivative is computed using the 3rd-order TVD Runge–Kutta algorithm.

Results

We elucidate model comparisons by computing the solution to an acoustically excited dilute bubble screen in water. The three-dimensional domain utilizes grid-stretching and non-reflective boundaries to minimize edge effects. The bubble screen occupies the central area of the domain and has uniform mesh spacing. Initially, the bubbles have a radius of $10\ \mu\text{m}$ and are randomly and homogeneously distributed in the screen region. A one-way plane acoustic source at excites a single cycle of a sinusoidal pressure wave towards the screen.

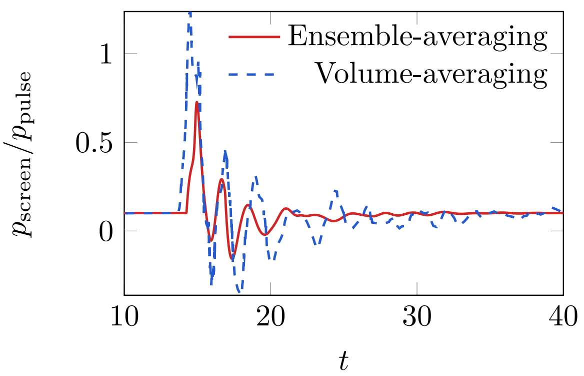

Figure 1: Mixture pressure at the center of the bubble screen. The acoustic pulse has frequency $300\ \text{kHz}$ and amplitude $1\ \text{MPa}$ and the bubble screen has void fraction 4×10^{-5} and length matching the wavelength of the pulse.

Figure 1 shows the pressure response of the central bubbly region. The models match qualitatively, though many instantaneous realizations of bubble distributions are required for the volume-averaged case to converge to the ensemble-averaged and, thus, statistically-averaged dynamics.

Further analysis

Statistical and computational trade-offs are computed and analyzed for the volume- and ensemble-averaged bubbly flow models. The relative cost between polydisperse computation and statistical convergence is analyzed and discussed.

Acknowledgments

The authors would like to thank Kevin Schmidmayer for fruitful discussions. This work was supported in part by the Office of Naval Research under grant N0014-17-1-2676.

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