Closure of phase-averaged bubbly, cavitating flow models

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Summary Phase-averaged bubbly flow models are closed by high-order statistical moments of the disperse bubble dynamics. Evaluating these moments in a simulation environment is computationally expensive because the integrands are highly oscillatory. The cost of this closure is demonstrated for an ensemble-averaged bubbly flow model. A machine-learning-based method for accelerating the moment evaluation is formulated.

INTRODUCTION

Bubble cloud dynamics play a central role in phenomena ranging from blast trauma injury[1] to kidney stone pulverization.[2] When the size distributions of bubble nuclei are broad, the average response of the bubbles to pressure fluctuations damps and disperses.[3] Bubbly flow models must account for such size distributions if they are to represent the dynamics of realistic bubbly flows.

PHASE-AVERAGING

Phase-averaged computational methods can represent the interaction between polydisperse bubble clouds and the flow that stimulates them. Current methods utilize either ensemble- or volume-averaging. Volume-averaging,[4] shown in figure 1 (left), represents bubbles as Lagrangian features that advect and evolve according to an assumed bubble dynamics model (e.g. Rayleigh–Plesset). The bubble population, sampled from an equilibrium bubble radius distribution, couples to the suspending liquid via a volumetric smoothing operator. Figure 1 (right) shows ensemble averaging.[5] It represents the bubble population as an evolving probability distribution, again according to a bubble dynamics model, in the equilibrium bubble size space on the Eulerian grid. The bubble population moments couple to the mixture-averaged flow equations and advect via a transport equation.

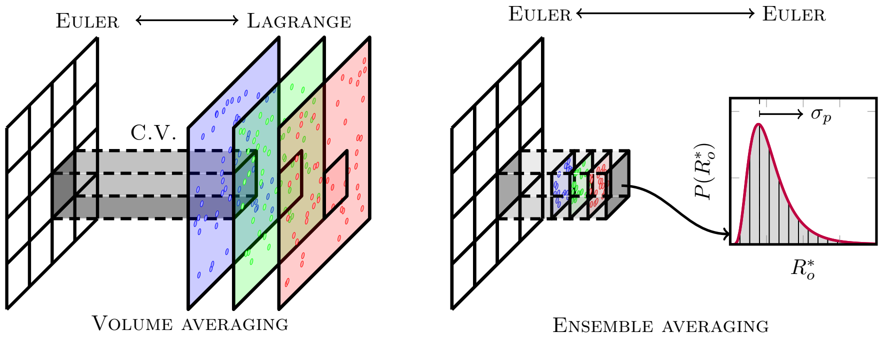

Figure 1: Schematic illustration of phase averaged methods.

Ensemble averaging is computationally cheaper than volume averaging when the bubble population is not significantly polydisperse.[6] However, ensemble averaging methods often approximate the required moments via classes methods. These methods bin and evolve the underlying probability density function. While simple, this approach is computational expensive for broad bubble size distributions. Our goal is to accelerate ensemble-averaged methods for polydisperse cases via a moment-based method.

CLOSURE VIA MOMENT METHODS

Statistical moments often more efficiently represent distributions than bins. Evolution of such moments follow from a population balance equation. This technique has been used to model polydisperse bubbly flows, though has not been applied to cavitating bubble populations, which undergo large volume changes. The moment evolution equations are not a function of only lower order moments when the bubble dynamics are nonlinear. Quadrature-based moment methods[7] can address this issue, though computing high-order moments with them is expensive. Instead, we use a Gaussian closure method. For this, a specific moments $\{l, m\}$ is

$$\mu_{lm} = \int R^l \dot{R}^m P(\vec{\mathbf{x}}, \vec{\boldsymbol{\theta}})\, \mathrm{d}\vec{\mathbf{x}} \tag{1}$$

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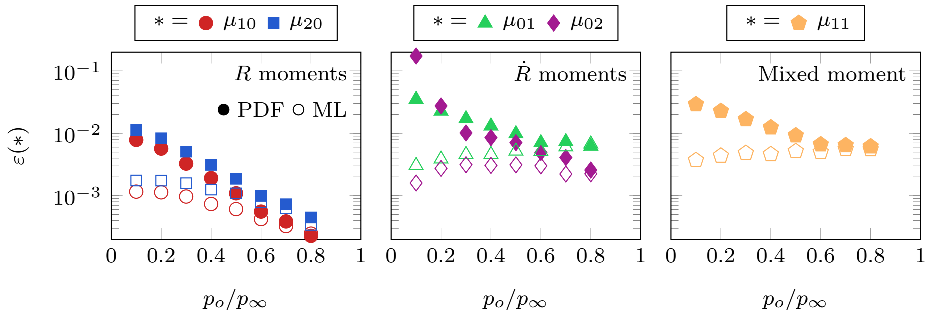

Figure 2: First- and second-order moment model errors. PDF and ML correspond to the Gaussian closure and neural network augmented methods, respectively.

where $\vec{\mathbf{x}} \equiv \{R, \dot{R}\}$ are the internal coordinates and $\vec{\boldsymbol{\theta}}$ are the shape parameters of the probability density function P (e.g. the means and variances). The moment system evolves as

$$\frac{\partial \mu_{lm}}{\partial t} = l\mu_{l-1,m+1} + m \int \ddot{R}(\vec{\mathbf{x}}) R^l \dot{R}^{m-1} P(\vec{\mathbf{x}}, \vec{\boldsymbol{\theta}})\, \mathrm{d}\vec{\mathbf{x}}. \tag{2}$$

The integral term of (2) is closed by assuming P is a bivariate Gaussian density function and that the Rayleigh–Plesset equation represents the bubble dynamics $\ddot{R}$ with driving pressure ratio p_o/p_∞. The moment system (2) is five dimensional because P has only two shape parameters. This is significantly smaller than the hundreds of bins that significantly polydisperse populations require.[6]

Figure 2 shows that the model error ε is small for all moments when the driving pressure ratio approaches unity. This is because the dynamics are linear in this limit and the Gaussian closure is exact. However, for lower pressure ratios the bubble dynamics are more violent and model errors are significant (figure 2 PDF, filled symbols). This model deficiency results from high-order moment development, which Gaussian closure does not represent. To treat this issue, we train a long short-term memory (LSTM) recurrent neural network (RNN) to augment the low-order moment evolution equation (2). Figure 2 (ML) shows that this significantly decreases the model error for validation cases at low pressure ratios.

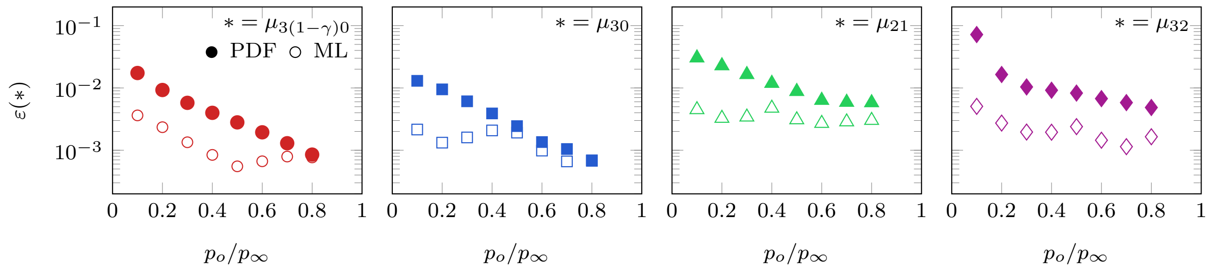

Figure 3: Model errors for the phase-averaged-model required moments.

Ensemble-averaged models require four high-order moments of the evolving population. Assuming Gaussian closure for P is sufficient to evaluate these, though figure 3 (PDF) shows that this again results in significant errors for low pressure ratios, even if the low-order moments are accurate. This is because the statistics are non-Gaussian in this regime. To treat this, we train another LSTM RNN to correct these moments. Figure 3 shows that this again significantly reduces the model errors, particularly for low pressure ratios.

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References

[1] K. Laksari, S. Assari, B. Seibold, K. Sadeghipour and K. Darvish, "Computational simulation of the mechanical response of brain tissue under blast loading," Biomech. Model. Mechanobiol. **14**, 459 (2015).

[2] Y. A. Pishchalnikov, O. A. Sapozhnikov, M. R. Bailey, J. C. Williams, R. O. Cleveland, T. Colonius, L. A. Crum, A. P. Evan and J. A. McAteer, "Cavitation bubble cluster activity in the breakage of kidney stones by lithotripter shockwaves," J. Endourol. **17**, 435 (2003).

[3] P. Smereka, "A Vlasov equation for pressure wave propagation in bubbly fluids," J. Fluid Mech. **454**, 287 (2002).

[4] L. V. Wijngaarden, "On the equations of motion for mixtures of liquid and gas bubbles," J. Fluid Mech. **33**, 465 (1968).

[5] D. Z. Zhang and A. Prosperetti, "Ensemble phase-averaged equations for bubbly flows," Phys. Fluids **6** (1994).

[6] S. H. Bryngelson, K. Schmidmayer and T. Colonius, "A quantitative comparison of phase-averaged models for bubbly, cavitating flows," Int. J. Mult. Flow **115**, 137 (2019).

[7] D. L. Marchisio and R. O. Fox, "Solution of population balance equations using the direct quadrature method of moments," J. Aerosol Sci. **36**, 43 (2005).