A UNIFIED EULERIAN MULTIPHASE FRAMEWORK FOR FLUID-STRUCTURE INTERACTION PROBLEMS INCLUDING CAVITATION

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Summary Understanding the impact load mechanisms from cavitation bubbles and shocks emitted by their collapse in and near solid deformable media is important for engineering and biomedical applications. In such flows, transient pressure fluctuations can lead to a cloud of small vapor bubbles near the solid object. A unified Eulerian framework for fluid-structure interaction problems including cavitation is developed to incorporate numerically unresolved and resolved bubbles and the solid material elasticity. The numerical model uses interface-capturing techniques for the fluid-structure coupling with phase change. The method is based on a high-order accurate weighted essentially non-oscillatory shock and interface capturing scheme. Studies of single bubble and cloud dynamics near a solid/compliant structure relevant to therapeutic ultrasound applications are presented.

INTRODUCTION

Understanding the bubble dynamics in or near compliant or hard matter is important for engineering and biomedical applications, particularly in the context of cavitation-induced damage. Biomedical applications include therapeutic ultrasound tools with ultrasound waves of amplitudes and frequencies ranging from -25 to 100 MPa and 100 kHz to MHz, respectively, to treat pathogenic tissues (soft) and stones (hard). Two examples of these tools are extracorporeal shockwave and burst wave lithotripsy, therapeutic ultrasound treatments using 4 to 40 MPa peak positive and -25 MPa peak negative pressure amplitudes and 100 kHz to MHz frequencies ultrasound waves, respectively, to break urinary stones (e.g., kidney and gall bladder stones). In burst wave lithotripsy, an array of ultrasound transducers emit focused ultrasounds waves that effectively fractionate larger stones due to the incident wave reflecting off the distal side of the stone switching sign and focusing to a maximum tensile stress. The negative pressure in the wave may lead to the generation of bubbles near the surface of the stone. The gas bubbles then respond to the burst wave by oscillating in volume, coalesce and forming larger bubbles, and/or collapse emitting shocks into the surroundings. The small stones have been shown to be most effective at eroding the stone.

To study these fluid-structure interaction problems, the challenge is to numerically simulate: (i) growth and coalescence of small bubbles in the bubble cloud to larger, resolvable bubbles and (ii) wave dynamics in the fluid(s) and nearby solid. During the oscillations, vapor can condense, gases can dissolve into the liquid, and vice versa across the bubble interface and affect the bubble dynamics. Unlike resolving a single bubble, resolving a population of bubbles in the bubble cloud is computationally unfeasible. These flows are typically modeled using a sub-grid model in an Eulerian framework [3, 4]. However, it is not well understood how to appropriately transition from a region in a bubble cloud where the unresolved (sub-grid) bubbles are sufficiently large to be resolved. Unlike the gas bubbles and surrounding liquid that lend themselves to an Eulerian framework, the nearby solid undergoes infinitely small to finite deformations that are well-suited to be captured in a Lagrangian framework. However, algorithmic complexity increases significantly with two separate solvers and coupling between them. Of the two current Eulerian approaches (i.e., hyperelastic (Godunov-based) and hypoelastic (conventional)) used to study wave dynamics and deformations in solids [2], the hypoelastic approach's algorithm is well-suited to incorporate elasticity to existing multiphase/multi-component numerical solvers [5]. Thus, we leverage the Eulerian numerical framework in the open-source Multi-component Flow Code (MFC) [1, 3, 8] in conjunction with (i) the heat and mass transfer relaxation approaches of [6, 7] and (ii) the hypoelastic numerical model of [5] to develop a numerical model and framework to solve multi-component fluid-structure interaction problems including multi-scale cavitation ranging from resolved singular bubbles to unresolved bubbles in a bubble cloud.

NUMERICAL MODEL AND METHODS

The open-source MFC solver, an Eulerian numerical shock- and interface-capturing method, is used to develop a unified numerical solver to solve 3D fluid-structure interaction problems with multi-scale cavitation. The solution is evolved in time using Runga-Kutta schemes. Approximate Riemann solvers (e.g., Harten-van Leer-Lax and contact, HLLC) in conjunction with a high-order accurate weighted essentially non-oscillatory schemes are used for upwinding the solution [1]. Bubble cloud model is an Eulerian-Eulerian sub-grid model [3] that resolves the cavitation bubble dynamics directly or models them via phase averaging, depending upon the characteristic length sizes and bubble number. The 6-equation numerical model of [6, 7] and associated relaxation procedures are solved to include heat and mass transfer (including phase change) at the material interfaces. As proposed by [5], evolution equations of the elastic contribution of the Cauchy stress tensor are solved to incorporate elasticity for material solid in the Eulerian numerical framework.

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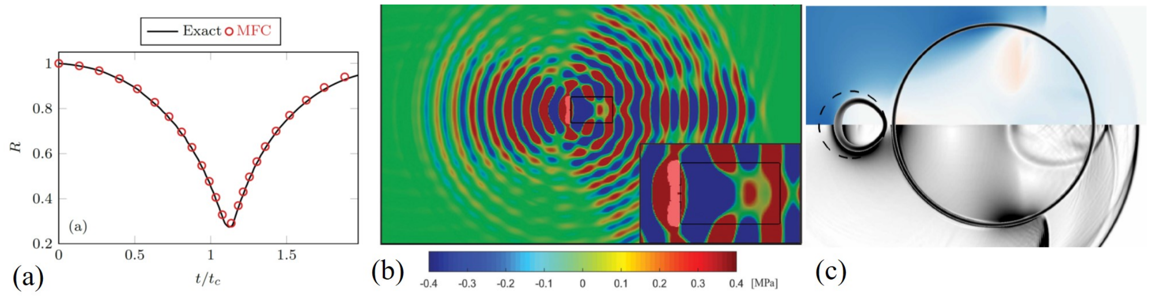

Figure 1: Cavitation bubble dynamics simulations: (a) validation of MFC with a resolved single spherical bubble collapse dynamics [8]; (b) pressure contours of MFC simulation of burst wave lithotripsy wave interacting with a cavitation cloud near a kidney stone (simulated as a stiff liquid) [4]; (c) Contours of pressure (top) and numerical Schlieren (bottom) of a single gas bubble collapsing near a spherical kidney stone using the hypoelastic approach of [5].

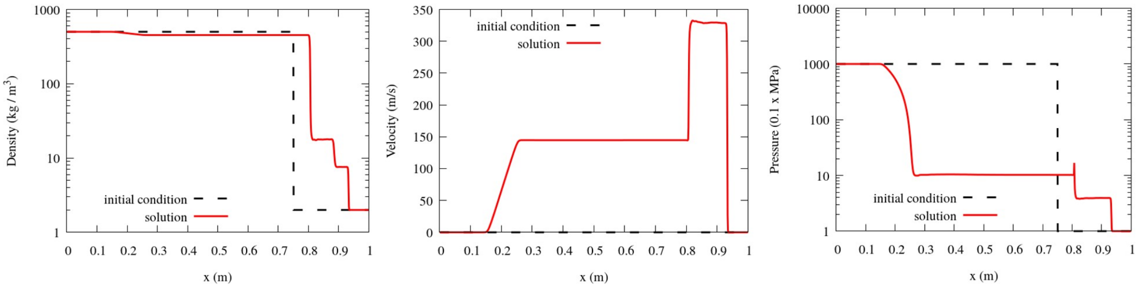

Figure 2: Dodecane liquid-vapor shock tube with mass transfer solved with MFC. The initial (black line) and numerical solution (red line) at $t = 400$ μs are shown.

RESULTS

A unified Eulerian numerical model and multiphase/multi-component framework for fluid-structure problems involving multi-scale cavitation has been developed using MFC. Figure 1 shows cavitation bubble dynamics simulations, the first two subfigures by MFC and third by the similar numerical framework of [5]. MFC has been validated for resolved, single spherical bubble dynamics (growth and collapse) with comparison to the numerical solution of the Keller-Miksis equation (see Fig. 1a) [8]. Numerical simulations of focused, burst wave lithotripsy experiments interacting with a bubble cloud at the surface of a cylindrical model kidney stone (represented by a stiff liquid) have been conducted in MFC to study the wave dynamics and damage mechanisms in the stone (see Fig. 1b) [4]. The numerical multiphase interface- and shock-capturing hypoelastic model of [5] and similar to MFC has been used to study the shock-induced single bubble collapse near a model spherical kidney stone (see Fig. 1c). The figure shows the transmission and internal reflection of the incident shock inside of the stone generating a focused region of tension (highlighted in red) inside of the stone as the bubble begins to collapse. Figure 2 shows the result of the dodecane liquid-vapor shock tube problem with mass transfer solved with MFC. The initial and numerical solutions at $t = 400$ μs are shown. A similar numerical solution of the density, showing the rarefaction, evaporation, contact and shock waves, velocity and pressure is also obtained by [7]. In addition to validation results, numerical simulations of fluid-structure problems involving the growth and collapse of a single bubble and/or bubble clouds with gas and vapor, undergoing phase change, in a liquid near a compliant or hard solid to understand the bubble and wave dynamics and associated damage mechanisms are also presented.

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